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Engineering Notes

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Performance Optimization for a Towed-Cable System with Attached Wind Sock

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Introduction

CIRCULARLY towed-cable systems have been studied for several decades with the goal of stabilizing the cable tip in inertial space [1–3]. When a cable is towed from an aircraft during a steady-banked turn, the cable end body tends to tow inside the path of the circle. As the cable length is increased and the cable end body weight to drag ratio is decreased, the cable tip approaches the center of the circle and ultimately appears to be nearly stationary when viewed from the inertial frame. The idea, proven successful in tests flights [4], is to use this fundamental property of the system for pickup and delivery of payloads [5].

In previous related work, the stability of equilibrium solutions has received significant focus [1–3,5–7]. Mathematically interesting phenomena occur for combinations of system parameters at high angular velocities. Recently, a lumped mass approximation for the cable dynamics was used to study the system [5]. Optimal system configurations were determined by taking into account the performance limitations of the towing aircraft and cable strength. To obtain nearly stationary motion of the cable end body, its steady-state orbit radius from the center of the aircraft orbit should be as small as possible. It has been observed both empirically and theoretically that increasing the drag properties of the cable end body, while simultaneously decreasing its weight, leads to smaller orbit radii. This fact suggests the following: what if another cable were to be deployed from a high-drag low-weight end body? Instead of actually deploying another cable, the idea pursued in this Note is to examine the system dynamics and equilibrium configurations with a high-drag device attached at some point along the cable. For the sake of simplicity, we refer to such a high-drag device as a "windsock," although any high-drag producing device could be employed. We use the methodology developed in [5] to study the effects of such a device in stabilizing the steady-state motion of the cable tip.

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Mathematical Model

Cable Model

The cable model used in this research is a lumped mass or lumped parameter model [8,9]. In the model, the cable is discretized into a series of point mass elements connected by elastic springs. All external forces, such as gravity and aerodynamic drag, are lumped to the point masses. This type of model is convenient and relatively simple to program. It also allows the rapid determination of relative equilibrium configurations, which are used extensively during optimization of the system. The accuracy of such models has been validated for a variety of towing conditions [10]. The major feature of the particular model used here is that the equations of motion are expressed in a rotating coordinate system attached to the aircraft. The cable takes up a relative equilibrium position when viewed from this frame. The wind sock is included in the model by introducing a pure drag force opposing the local flow at the sock's position. The sock mass is also added to the lumped mass (or masses) closest to its position. The atmospheric density is assumed to follow the international standard atmosphere, and the lift and drag coefficients of the cable are taken from [11]. More detail on the model can be found in [5].

Equilibrium Configurations and Their Stability

Equilibrium, or quasi-stationary solutions, are obtained using the lumped mass model and a shooting method. That is, by specifying the position of the cable end body, it is possible to determine all forces acting on it except the cable tension. This gives the magnitude and direction of the tension force necessary to maintain the end body position. Similarly, given a cable element length, it is possible to determine the elemental strain and hence the position of the next discrete mass. This allows all forces on the lumped mass to be determined except the tension from the next cable element. This process continues up the entire cable until the aircraft altitude is reached. If the orbit radius of the aircraft is specified, then the position of the end body is iterated on until the cable attachment coincides with the aircraft.

The equilibrium solution is used as a reference to compute a set of linearized perturbed differential equations via a central finite difference method. The resulting Jacobian matrix is used to calculate the linear stability characteristics of the system by solving the associated eigenvalue problem. The eigenvalues define the natural frequencies and damping ratios of the system for the particular configuration being studied.

Optimization Methodology

For the system to be physically realizable, the tow-point motion must satisfy particular constraints dictated by the performance limitations of the aircraft. The optimization problem is posed as follows: Find the aircraft orbit radius r, its angular velocity ω, and the cable diameter d to minimize the cost function

$$\mathcal{J} = r_d^2 \tag{1}$$

subject to the constraints

$$d \geq \sqrt{\frac{4|\boldsymbol{T}|_{\max}}{\pi\sigma_{\text{ut}}}}, \qquad \phi \leq \phi_{\max}, \qquad C_L \leq C_{L_{\max}}, \qquad n \leq n_{\max}$$
$$P \leq P_{\max} \tag{2}$$

where r_d is the orbit radius of the cable end body, $|\boldsymbol{T}|_{\max}$ is the maximum tension force in the cable, σ_{ut} is the ultimate tensile strength of the cable, ϕ is the aircraft bank angle, C_L is the lift coefficient, n is the load factor, and P is the aircraft power. In this Note, optimal configurations are generated using the following system properties: cable end mass = 50 kg, cable length = 3000 m, end body drag (drag coefficient × drag area) $C_D A_d = 2\ \text{m}^2$, $\phi_{\max} = 70$ deg, $C_{L_{\max}} = 1.1$, and $n_{\max} = 4$. The resulting optimization problem is solved using the sequential quadratic programming software SNOPT [12].

Numerical Study of Wind-Sock Cable System

In [5], three types of aircraft were studied in relation to a cable system without any wind-sock attachment. However, it was found that a light aircraft provides the best capabilities for obtaining a small end body orbit radius, and so only a light aircraft configuration is considered here. The aircraft parameters are given by aircraft mass = 2000 kg, wing span = 9 m, wing area = 20 m^2, and maximum power = 300 hp. The drag coefficient of the wind sock is assumed to be 1.35, and its diameter is 5 m (a large sock). The cable material is assumed to be spectra with a density of 970 kg/m^3 and Young's modulus of 120 GPa.

An unmodified system, that is, one without any wind-sock attachment, was also optimized using the above methodology with the cable discretized into 200 elements. The optimal system is characterized by an orbit radius of the cable end body of 3.536 m and an aircraft orbiting at a radius of 220.93 m at a speed of 51.45 m/s. The required cable diameter is 2.36 mm. The first three nondimensional natural frequencies of the system (nondimensionalized with respect to the aircraft angular velocity) are 0.676, 0.983, and 1.172, and the corresponding damping ratios are 0.282, 0.087, and 0.520. The same system was optimized with a wind sock attached at distances between 5 and 90% of the cable length from the bottom of the cable. The results from the numerical optimization are shown in Figs. 1 and 2.

Figure 1 shows that the aircraft orbit radius and corresponding velocity are basically unchanged by the presence of the wind sock. The required orbit radius of the aircraft tends to decrease as the wind sock is positioned closer to the aircraft. At a distance of 10% of the cable length from the aircraft, the required orbit radius decreases by approximately 5.04 m or 2.28% of the radius with no sock. Conversely, the required aircraft velocity increases as the sock moves towards the aircraft. At a distance of 10% of the cable length from the aircraft, the aircraft velocity is 0.78 m/s or 1.52% faster than the system with no sock. The main effect of the wind sock is seen in Fig. 2, which shows the required cable diameter and orbit radius of the end body. The presence of the wind sock actually reduces the maximum cable tension relative to the case with no sock, and hence, the required cable diameter and cable mass are reduced. There appears to be an optimum location roughly midway along the cable where the diameter is reduced by 1.13 mm or 47.7% of the diameter required without a sock. Figure 2 also shows that this minimum is maintained over a wide range of sock positions (roughly 20% of the cable length), and hence, exact positioning of the sock is not necessary to achieve this improved performance. The advantage of this solution is that it also corresponds to the minimum orbit radius of the cable end body: 1.045 m for a sock at 50% of the cable length. This is a reduction of 70.4% from the steady-state radius achieved without the wind sock. The reduction in velocity of the cable end body is on the order of 0.6 m/s for this case.

Figure 3 shows the natural frequencies and damping ratios for the fundamental cable modes. Comparing the results in Fig. 3 with the frequencies of the unmodified system suggests that the presence of the sock has a significant effect on the fundamental frequencies of the system but that the effect depends on the sock's position. For example, a sock positioned at 15% of the cable length from the bottom reduces the fundamental frequency by nearly 38%, whereas a sock at 55% of the cable length reduces the frequency by 0.45%. As the sock moves towards the aircraft, the first and third frequencies both increase, whereas the second frequency decreases. The position of the sock also causes some substantial changes in the damping ratios of the fundamental modes. In all cases the system is stable, but for a sock positioned at the cable midpoint (optimum position), the damping of the first mode is reduced by 74.3% and the damping of the second mode is reduced by 51.2%. Hence, disturbances to the system due to winds or pilot error would take longer to damp out.

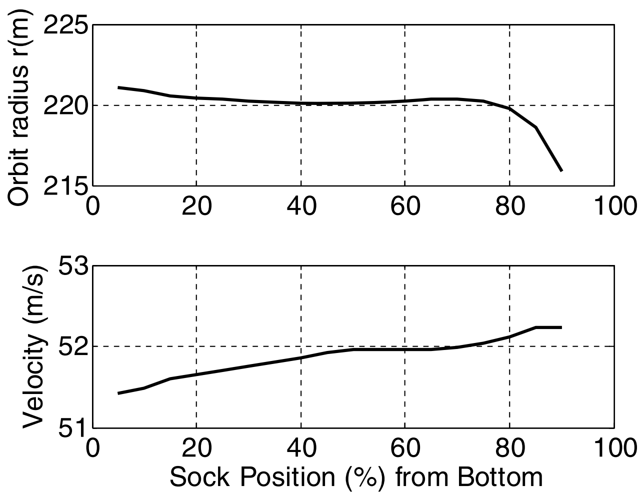


Fig. 1 Optimal orbit radius and aircraft velocity.

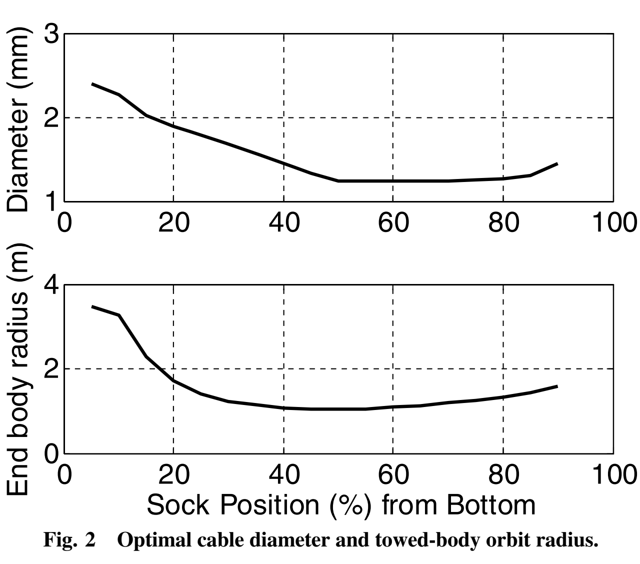


Fig. 2 Optimal cable diameter and towed-body orbit radius.

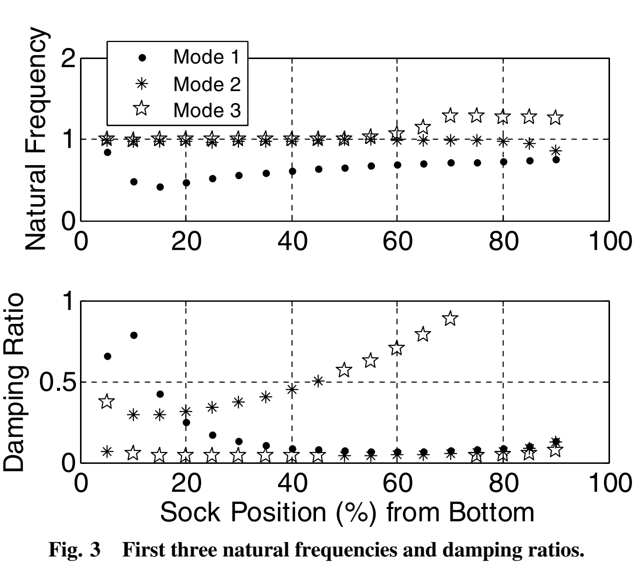
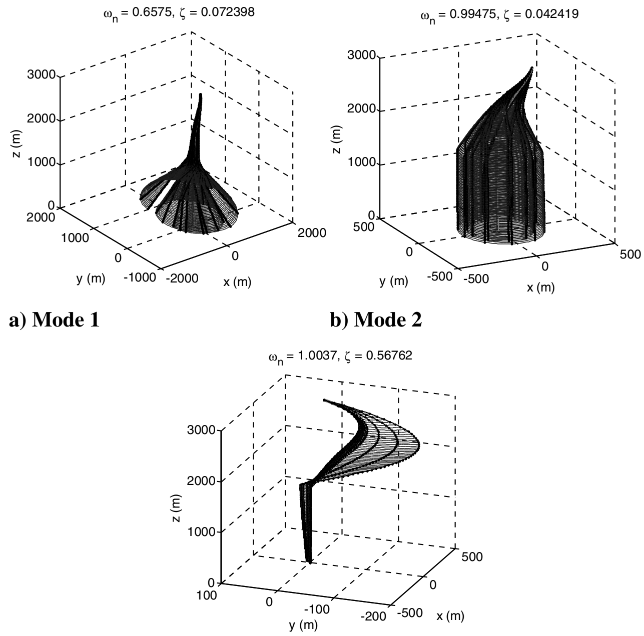


Fig. 3 First three natural frequencies and damping ratios.



a) Mode 1 **b) Mode 2**

c) Mode 3

Fig. 4 Examples of first three modes with wind sock attached at cable midpoint.

Figure 4 shows the mode shapes of the fundamental modes (viewed in the rotating coordinate system) for the wind-sock system with the sock at the cable midpoint. The first mode is a spiraling pendulum-type mode of the end body, with a pivot created by the wind sock. The second mode is a spiraling pendulum-type mode of the cable with rotation around the aircraft attachment. As illustrated in Fig. 4, the first two modes directly affect the position of the cable tip. The third mode is a lateral string mode of the portion of the cable above the wind sock. Note that the wind sock essentially decouples the motion of the two halves of the cable in the first and third modes. Thus, the improved static performance of the wind-sock system comes at the price of reduced dynamic performance. Because the goal is to stabilize the cable tip, disturbances could make the system with a wind sock more difficult to control and position accurately.

Conclusions

The static performance of a circularly towed-cable system with a high-drag device attached at a point along the cable has been optimized using numerical techniques. A high-drag device positioned at the cable midpoint is able to reduce the cable end body radius by 70.4% and reduce the inertial velocity by 0.6 m/s without requiring any substantial changes to the aircraft orbit. Furthermore, the maximum cable tension is reduced such that the required cable diameter is approximately 52% of that required for the case without a high-drag device. However, the increased static performance is offset by a reduction in the dynamic performance. The first two modes of the system, apart from being slightly reduced in frequency, are significantly reduced in damping. These modes dominate the dynamic positioning of the cable tip, and hence, the wind-sock system would most likely be harder to stabilize in the presence of disturbances.

Acknowledgment

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References

[1] Skop, R. A., and Choo, Y.-I., "The Configuration of a Cable Towed in a Circular Path," *Journal of Aircraft*, Vol. 8, No. 11, 1971, pp. 856–862.

[2] Russell, J. J., and Anderson, W. J., "Equilibrium and Stability of a Circularly Towed Cable Subject to Aerodynamic Drag," *Journal of Aircraft*, Vol. 14, No. 7, 1977, pp. 680–686.

[3] Clifton, J. M., Schmidt, L. V., and Stuart, T. D., "Dynamic Modeling of a Trailing Wire Towed by an Orbiting Aircraft," *Journal of Guidance, Control, and Dynamics*, Vol. 18, No. 4, 1995, pp. 875–881.

[4] Hitt, R. T., *Jungle Pilot*, Discovery House Publishers, Grand Rapids, MI, 1997.

[5] Williams, P., and Trivailo, P., "Dynamics of Circularly-Towed Aerial Cable Systems—Part I: Optimal Equilibrium Configurations and Their Stability," *Journal of Guidance, Control, and Dynamics* (to be published).

[6] Zhu, F., and Rahn, C. D., "Stability Analysis of a Circularly Towed Cable-Body System," *Journal of Sound and Vibration*, Vol. 217, No. 3, 1998, pp. 435–452.

[7] Russell, J. J., and Anderson, W. J., "Equilibrium and Stability of a Whirling Rod-Mass System," *International Journal of Non-Linear Mechanics*, Vol. 12, No. 2, 1977, pp. 91–101.

[8] Leonard, J. W., and Nath, J. H., "Comparison of Finite Element and Lumped Parameter Methods for Oceanic Cables," *Engineering Structures*, Vol. 3, No. 3, 1981, pp. 153–167.

[9] Kamman, J. W., and Huston, R. L., "Modeling of Variable Length Towed and Tethered Cable Systems," *Journal of Guidance, Control, and Dynamics*, Vol. 22, No. 4, 1999, pp. 602–608.

[10] Buckham, B. J., and Nahon, M., "Formulation and Validation of a Lumped Mass Model for Low-Tension ROV Tethers," *International Journal of Offshore and Polar Engineering: Transactions of the ISOPE*, Vol. 11, No. 4, 2001, pp. 282–289.

[11] Cochran, J. E., Innocenti, M., No, T. S., and Thukral, A., "Dynamics and Control of Maneuverable Towed Flight Vehicles," *Journal of Guidance, Control, and Dynamics*, Vol. 15, No. 5, 1992, pp. 1245–1252.

[12] Gill, P. E., Murray, W., and Saunders, M. A., "SNOPT: An SQP Algorithm for Large-Scale Constrained Optimization," *SIAM Journal on Optimization*, Vol. 12, No. 4, 2002, pp. 979–1006.